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Turbine-Generator System Control for a HTGR Power Plant*

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Linear multivariable systems analysis techniques and nonlinear simulation, used to investigate system dynamic behavior and governor design for the dual turbine-generator system of a prototypical 1150 MW HTGR power plant, provide essential information for practical large scale multivariable system designs.

Key Words—Control system analysis; dynamic response; electric generators; load regulation; speed control; steam turbines.

Abstract—Dual turbine-generator systems offer several operating advantages for large electric power generating plants. In order that these benefits be fully realized, it is necessary that the twin turbine-generator system, including its associated control systems, perform satisfactorily in a variety of normal and energy operating modes. This paper combines nonlinear simulation and linear multivariable systems analysis techniques to investigate system dynamic behavior and governor design. Specific results are reported for load following operation in which one turbine-generator is shut down.

1. INTRODUCTION

THE USE of dual turbine-generator systems in large nuclear power plants is an option being selected for a number of new installations. For the High Temperature Gas-Cooled Reactor (HTGR), in particular, with its favourable steam conditions, the choice of using two half-size machines as opposed to a single full size machine involves little penalty, whereas utility operating experience indicates significant advantage in reliability and availability (Gibbons, Waage and Sieving, 1972). In order that the potential benefits by fully realized from the operational flexibility afforded by the dual configuration, it is necessary that the twin turbine-generator system and its associated controls be capable of satisfactory performance in several distinct operating modes. In addition to normal, equally loaded operation, the plant must be capable of operating with only one turbine, trip of one turbine while the other continues to operate, and startup of an idle turbine while the other is in operation (Jaegtnes, McDonald and Broer, 1974). Moreover, such installations are generally characterized by relatively strong coupling between the turbine-generator sets of both an electrical and thermofluid-dynamic nature and frequently exhibit strong coupling with the reactor itself. In view of this, a comprehensive understanding of the dynamics of such systems is desirable and is the motivation for the studies reported herein.

In this paper a dual turbine-generator system for a prototypical 1150 MW HTGR power plant is considered. Such a system has been the subject of an extensive simulation study (Broer and co-workers, 1974) (involving Drexel University, General Atomic Company, Philadelphia Electric Company, Stone and Webster Corp. and Westinghouse Electric Company) with the primary objective of investigating overall plant behavior for both steady-state and dynamic conditions of interest during startup, shutdown, normal and transient operation. In addition, independent simulation studies have been conducted (Jaegtnes, McDonald and Broer, 1974) by Westinghouse Electric Company with the main purpose of investigating the dynamic interaction between the two turbine-generators, between the helium circulator turbines and the turbine generators, and between the steam headers and the turbine generators as well as for the evaluation of turbine control systems.

The study described herein is based upon linearization and the application of linear multivariable control system design techniques. There are four primary objectives to this investigation:

to investigate controllability and observability,
to formulate dynamic control objectives,
to evaluate alternative measurement sets,

such an analysis can be a valuable adjunct to simulation studies.

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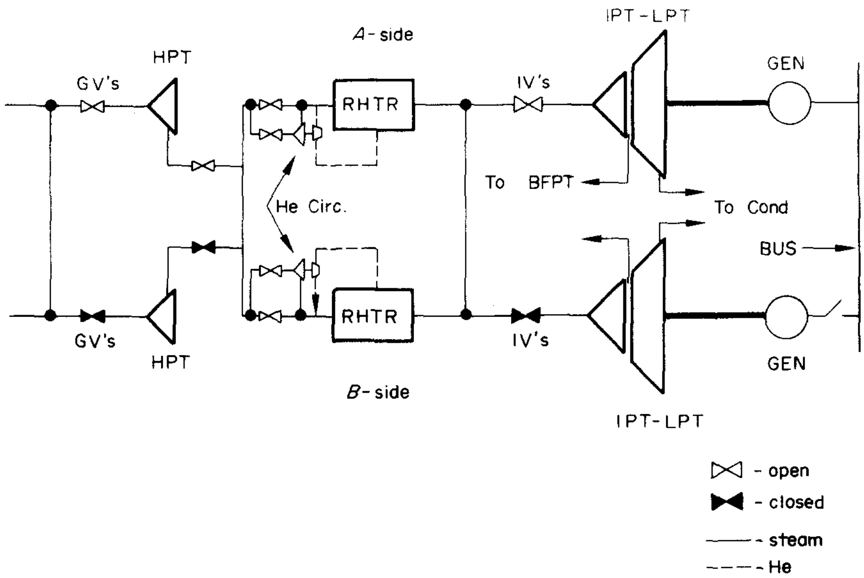

FIG. 1. Twin turbine generator model schematic.

The investigations reported in this paper are concerned with the unbalanced alignment in which one turbine-generator set is shut down and the system is in a load following mode with the available turbine-generator operating in the upper 50% of its load range. In this restricted situation only governor valve stroke is used for control, however, any subset of the process variables—speed, generation and first stage pressure—may be monitored. Consequently, the designer is faced with a single-input–multiple-output control problem. The analysis is facilitated by the use of multivariable techniques including the calculation of multivariable system zeros.

The model has been linearized and a modal analysis conducted at a load level corresponding to 75% of maximum turbine load, the results of which are discussed in Section 3, and 4. It is noted that several stable, but rather slow, modes involving thermodynamic variables are uncontrollable and unobservable with respect to the usual control and measurement sets. This is an important observation with regard to the formulation of control objectives and the selection of a control design procedure which is the subject of Section 5.

A primary objective of the paper is to evaluate alternative choices for output sets. The control objective, formulated in Section 5 is to regulate a linear combination of speed error and generation error to zero while providing closed-loop system time constants of 20 s or less for selected modes. Control systems are designed in Section 5, using different output sets to meet this common control objective. For this purpose all controllers are designed using the same state variable design methodology which incorporates an observer and constant disturbance accommodation. In all cases, the design is based on a reduced order linear model which retains only those modes corresponding to eigenvalues of magnitude less than or equal to 0.25. Evaluations of alternative controllers are made on the basis of comparison of load following and disturbance rejection properties. These are examined by analysis of open and closed loop pole-zero patterns and by simulation using the nonlinear model. For the latter, system response to generation demand changes are evaluated and also response to switching from partial arc to full arc admission.

The results are summarized in Section 6.

2. SYSTEM DESCRIPTION AND MATHEMATICAL MODEL

The basic plant structure is as follows. Steam is generated in six parallel helium-heated once-through steam generators and upon leaving the steam generators flows into a common main steam header which supplies the two high pressure turbines through the turbine valves and control valves. The high pressure turbines exhaust into a common exhaust header which supplies steam through cold reheat headers to six turbines which drive the helium circulators. Exhaust steam from each circulator turbine then passes through a reheater before entering a common hot reheat header. The hot reheat header supplies steam to the two intermediate pressure turbines each of which exhausts through crossover piping to two low pressure turbines which in turn, exhaust to the condenser (Gibbons, Waage and Sieving, 1972 and Broer and coworkers, 1974).

During the normal load following operation the plant has four primary control loops which regulate: 1) generation, 2) main steam pressure, 3) main steam temperature and 4) reheat temperature. This is accomplished by a coordinated control structure with the following control signals being in principal correspondence with the four regulated variables: 1) governor valve position demand, 2) feedwater valve position demand, 3) helium circulator speed demand, and 4) reactor power level demand.

In this paper the generation control loop is under study and the process is modeled only from the main steam header through to the condenser on the steam side. It is assumed that main steam header conditions are constant during the time interval of interest (from a few seconds to one hundred seconds). This is tantamount to assuming an extremely fast and well coordinated pressure control system and also that the temperature dynamics are naturally so slow that the temperature will not vary significantly during the time span of interest. The latter assumption is probably quite reasonable (Hastings and Louis, 1971). However, there is typically quite a bit of interaction between the pressure and generation control loops and the analysis cannot be considered complete until overall operation is evaluated by a comprehensive simulation. Nevertheless, the constant pressure assumption is reasonable for a preliminary investigation.

There are a number of secondary control systems which are involved with that portion of the plant of interest. These are the circulator speed and pressure ratio controllers and also the generator voltage regulators (exciters). Of these, only the voltage regulators are considered active and included in the model. The mathematical model used to represent the process is actually a combination of several component models:

High Pressure Turbine*
Turbine Exhaust and Cold Reheat Headers
Helium Circulator Turbine and Compressor
Reheater*
Hot Reheat Header
Intermediate and Low Pressure Turbines*
Electrical Generator*

The starred (*) component models were developed by the authors and their colleagues at Drexel University while the others were supplied by Philadelphia Electric Company. All were developed as part of the aformentioned HTGR simulation program. Each component model was derived from first principal considerations, i.e., derivations were based upon the physical laws of conservation of mass, energy, and momentum. Plant parameters such as turbine efficiencies, valve characteristics, operating conditions, etc. were either chosen on the basis of data obtained from corresponding equipment in existing plants or otherwise estimated.

The nonlinear process model contains thirty-eight (38) first-order differential equations and many more algebraic equations and takes the form:

$$\dot{x} = f(x, y, u) \tag{1a}$$

$$0 = g(x, y, u) \tag{1b}$$

where the 38 variables, x, are listed in Table 1, the single control input, u, is governor valve stroke and the outputs, y, include electric generation, frequency, impulse chamber pressure, governor valve flow, cold reheat temperature and many others. The model was coded for digital simulation using the Digital Simulation Language, (DSL). Coding for the version of DSL employed is almost identical to IBM's Continuous System Modeling Program (CSMP) and, in fact, this model can be run on most versions of CSMP with, at most, minor adjustments.

3. LINEARIZATION AND MODAL ANALYSIS

As has been indicated, the analyses to be performed herein requires linear process models. To facilitate the linearization process, an algorithm was derived to numerically calculate the linear system. The following is a brief description as to how the algorithm functions.

First, consider the linearization of (1) about the steady-state point $(\bar{x}, \bar{u})$. Calculation of the Jacobian matrices from this form leads directly to

$$\dot{x} = \bar{A}x + \bar{B}u + T_1 y \tag{2a}$$

$$0 = \bar{C}x - \bar{D}u + T_2 y. \tag{2b}$$

In order to obtain the desired standard form:

$$\dot{x} = Ax + Bu \tag{3a}$$

$$y = Cx + Du \tag{3b}$$

it is necessary to invert T_2 which may be large albeit sparse. Inversion of large matrices is both tedious when done analytically and 'noisy' when attempted numerically. Such inversion can be avoided and the linearization process actually simplified.

TABLE 1. PROCESS MODEL STATE VARIABLES

No.	Name	Description (Nominal Value at 75% Load)
1	RHP1A	Impulse Chamber Density (2.000 lb/ft^3)
2	UHP1A	Impulse Chamber Internal Energy (1280 btu/lb)
3	RHX	HP Exhaust Header Density (1.200 lb/ft^3)
4	HHX	HP Exhaust Header Average Enthalpy (1351 btu/lb)
5	HHXA	HP Exhaust Header Enthalpy (A-Side) (1351 btu/lb)
6	HHXB	HP Exhaust Header Enthalpy (B-Side) (1351 btu/lb)
7	RCRA	Cold Reheat Header Density (A-Side (1.194 lb/ft^3)
8	HCRA	Cold Reheat Header Enthalpy (A-Side) (1352 btu/lb)
9	RCRB	Cold Reheat Header Density (B-Side) (1.194 lb/ft^3)
10	HCRB	Cold Reheat Header Enthalpy (B-Side) (1352 btu/lb)
11	HCTA	Helium Circ. Turbine Inlet Enthalpy (A-Side) (1351 btu/lb)
12	RCTA	Helium Circ. Turbine Inlet Density (A-Side) (0.8614 lb/ft^3)
13	HCTB	Helium Circ. Turbine Inlet Enthalpy (B-Side) (1351 btu/lb)
14	RCTB	Helium Circ. Turbine Inlet Density (B-Side) (0.8614 lb/ft^3)
15	NCTA	Helium Circ. Turbine Speed (A-Side) (227.2 rpm)
16	NCTB	Helium Circ. Turbine Speed (B-Side) (227.2 rpm)
17	HATA	Attemperator Spray Enthalpy (A-Side) (1348 btu/lb)
18	HATB	Attemperator Spray Enthalpy (B-Side) (1348 btu/lb)
19	UARH	Reheater Internal Energy (A-Side) (1377 btu/lb)
20	RHOARH	Reheater Density (A-Side) 0.5427 lb/ft^3)
21	TMARH	Reheater Metal Temperature (A-Side) (1500 °R)
22	UBRH	Reheater Internal Energy (B-Side) (1377 btu/lb)
23	RHOBRH	Reheater Density (B-Side) 0.5427 lb/ft^3)
24	TMBRH	Reheater Metal Temperature (B-Side) (1500 °R)
25	RHR	Hot Reheat Header Density (0.5338 lb/ft^3)
26	HHR	Hot Reheat Header Average Enthalpy (1538 btu/lb)
27	HHRA	Hot Reheat Header Enthalpy (A-Side) (1538 btu/lb)
28	HHRB	Hot Reheat Header Enthalpy (B-Side) (1538 btu/lb)
29	LIV1A	Intercept Valve Lift (1.000 p.u.)
30	RIP1A	IP Turbine First Stage Density (0.5200 lb/ft^3)
31	UIP1A	IP Turbine First Stage Internal Energy (1377 btu/lb)
32	RLP1A	LP Turbine Inlet Density (0.1847 lb/ft^3)
33	ULP1A	LP Turbine Inlet Internal Energy (1294 btu/lb)
34	NGNEA	Turbine-Generator Speed Error (0.01896 rad/sec)
35	AGNIA	Rotor Angle (1.106 rad)
36	NEXFA	Exciter Field Voltage (1.807 p.u.)
37	JEXVIA	Stabilizer State Variable (0.1447 p.u.)
38	VEXSA	Excitation Voltage (1.811 p.u.)

(Exciter base 253.2 V)

To begin with, it is noted that implementation of the DSL Simulation phase requires the resolution of (1b) into the form

$$\hat{y}_i = \hat{g}_i(x, u, \hat{y}_1, \ldots, y_j, t) j < i \quad (4)$$

$$\text{for} \quad i = 1, 2, \ldots, n + r$$

$$n = \dim(x), r = \dim(y)$$

where $\hat{y}_i$ is the ith component of vector $\hat{y}$ which results upon forming the concentration of y and x of (1). Obtaining such a set of equations may involve solving several disjoint subsets of nonlinear algebraic equations along with establishing an appropriate ordering of the elements of $\hat{y}$. Simulation languages such as DSL, CSMP and others provide a useful aid in this regard via their sorting process. Thus, the derivation of (4) is the analogue of inverting T_2 in (2).

It is the main purpose to compute the matrices of (3). These can be numerically calculated via proper interfacing between DSL and a differentiating routine. This circumvents the need for subsequent inversion of T_2. Richardson's and Romberg's extrapolation method to successively computed central divided differences is used for these computations (Fillipi and Engels, 1966).

The nonlinear model was linearized about an operating point approximating the 75% maximum turbine load operating conditions and the eigenvalues of the 38 modes are listed in Table 2. Also included in the Table is information which provides an indication of the degree of controllability and observability. This data is obtained by transforming the system into diagonal form. The number reflecting 'degree of controllability' of each mode is simply the order of magnitude of the corresponding element of the transformed B matrix after the single column has been normalized so that its largest element has order of magnitude of one. Similarly, the 'degree of observability' data for each mode, with respect to each of the three measured outputs (frequency, generation, impulse chamber) is the order of

TABLE 2. TABULATION OF SYSTEM MODES

Mode	Eigenvalue	Degree of controllability† and observability‡			
		Valve Lift	Freq.	Gen.	Pres.
1	0.0	*	*	*	*
2	0.0	*	*	*	*
3	−0.0073	10^{-2}	*	*	*
4	−0.0479+j0.119	10^{-1}	*	1	10^{-2}
5	−0.0479+j0.119	10^{-1}	*	1	10^{-2}
6	−0.0619	*	*	10^{-2}	*
7	−0.132+j0.00804	10^{-1}	*	10^{-1}	*
8	−0.132−j0.00804	10^{-1}	*	10^{-1}	*
9	−0.154	*	*	10^{-1}	*
10	−0.212	10^{-2}	*	10^{-1}	10^{-2}
11	−0.224	10^{-1}	*	10^{-1}	10^{-2}
12	−0.274	10^{-2}	*	10^{-1}	*
13	−0.284	10^{-2}	*	1	*
14	−0.305	*	*	10^{-1}	*
15	−0.305	10^{-2}	*	10^{-2}	*
16	−0.451	10^{-1}	*	*	*
17	−0.338+j0.602	10^{-1}	10^{-2}	10^{-2}	*
18	−0.388−j0.602	10^{-1}	10^{-2}	10^{-2}	*
19	−1.38	10^{-1}	*	*	*
20	−1.45	10^{-2}	10^{-2}	10^{-1}	*
21	−1.48	10^{-2}	10^{-2}	*	*
22	−1.56	*	10^{-2}	10^{-1}	*
23	−2.02	10^{-2}	10^{-2}	10^{-1}	*
24	−2.71	10^{-2}	10^{-1}	10^{-1}	*
25	−3.27	*	10^{-1}	10^{-1}	*
26	−0.362+j4.56	1	10^{-1}	1	*
27	−0.362−j4.56	1	10^{-1}	1	*
28	−5.43	*	10^{-1}	10^{-1}	*
29	−7.62	*	10^{-1}	10^{-1}	*
30	−7.62	*	10^{-2}	10^{-2}	*
31	−8.40	*	*	*	*
32	−10.8	1	1	1	10^{-1}
33	−30.6	10^{-1}	10^{-2}	10^{-1}	*
34	−38.2	1	1	10^{-1}	1
35	−82.2	*	10^{-2}	*	*
36	−117	*	*	*	*
37	−120	*	10^{-2}	*	*
38	−193	*	10^{-1}	10^{-2}	10^{-1}

*Less than or equal to 10^{-3}.

†Order of magnitude of the element in the corresponding row of the transformed B-matrix (normalized).

‡Order of magnitude of the element in the corresponding row and column of the transformed C-matrix (normalized).

magnitude of the element in the corresponding column (mode) and row (output) of the transformed C matrix after each row has been normalized so that the order of magnitude of its largest element is one. Brief physical descriptions of selected modes are included in Table 3.

4. MODEL REDUCTION AND PROCESS ZEROS

The eigenvalues listed in Table 2 exhibit the presence of dynamic modes in a wide frequency range—from very slow to extremely fast. Based on this observation, the alternative of using a reduced order model—extracted from the 75% load model—for further analysis and controller–estimator design was chosen. The reduction of order was achieved by including only those modes corresponding to eigenvalues of magnitude less than or equal to 0.25. This number was chosen in accordance with the typical sampling rate for a digital governor of one (1) second. Such a rate restricts regulation to modes corresponding to time-constants at least as large as five (5) seconds. Thus, the choice of 0.25 for limiting eigenvalue magnitude yields a reduced model with dynamic modes safely in the region of interest.

The states of the reduced system are related to the original system states through a transformation that has not been included because of its size.

The slowest three modes listed in Table 2 are

TABLE 3. DESCRIPTION OF SYSTEM MODES

1	Intercept valve drive motor
2	Energy balance in hot reheat header
3	Energy balance in HP turbine exhaust header
4 & 5	Complex oscillatory modes involving mass and energy transfer between circulator turbine inlet volumes and front and back ends of main turbine
6	Mass and energy transfer between A and B side reheaters
7 & 8	Another pair of complex oscillatory modes involving mass and energy transfer between front and back ends of main turbine with nodes at the reheaters
9	Mass and energy transfer between storage elements just upstream and downstream of reheaters
10	Mass and energy transfer between storage elements in front end of turbine through cold reheat headers and back end of turbine including hot reheat header and crossover piping
11	Mass and energy transfer from HP turbine exhaust and cold reheat headers to circulator turbine inlet and hot reheat headers (or vice versa)
17 & 18	Exciter induced rotor oscillation
26 & 27	Synchronizing oscillation (with infinite bus)
32	Energy balance in impulse chamber and HP turbine exhaust head with resultant effect on electrical power
33	Exciter
34	Similar to 32
38	Mass and energy transfer involving storage elements in front end of turbine (HP turbine exhaust header to impulse chamber and cold reheat headers)

simultaneously unobservable and uncontrollable and consequently have been eliminated from the model.

The single control force is the aggregate governor valve lift, LGVA. The measurements considered are:

1. NGNEA Frequency deviation
2. PGNEPA Electrical generation
3. PHPl Impulse Chamber pressure

Note that in the discussions below the single output case refers to a linear combination of frequency deviation and generation error referred to as unit control error. The two output case employs frequency deviation and generation, and the three output case employs frequency deviation, generation and pressure.

The system zeros have been calculated (Kalnitsky and Kwatny, 1977) using the reduced model. Upon examination of Tables 4 and 5 it is observed that for the single output case pole-zero cancellation occurs (or nearly so) for the three modes with eigenvalues -0.619×10^{-1}, -0.154, -0.212. These correspond to modes numbered 6, 9, and 10, respectively in Table 2. Note, that modes 6 and 9 are uncontrollable. Mode 10 is

TABLE 4. EIGENVALUES OF REDUCED OPEN-LOOP PLANT

-0.479×10^{-1}	$\pm j0.119\times10^{-1}$
-0.619×10^{-1}	
-0.132	$\pm j0.804\times10^{-2}$
-0.154	
-0.212	
-0.224	

only weakly controllable and weakly observable in generation and unobservable in frequency, the two elements of the first output.

In the two output case all of the system zeros disappear except the three discussed above. With the addition of impulse chamber pressure—the three output case—the zero corresponding to mode 10 is eliminated.

The non-minimum phase zero of the single output case warrants some discussion. It occurs because in the reduced model a step increase in valve position produces an instantaneous drop in frequency which, in turn produces a drop in unit control error. In steady-state, of course, frequency error returns to zero and the generation change is positive resulting in a positive unit control error. Thus, the single output, unit control error, initially moves in a direction opposite to its

TABLE 5. REDUCED OPEN LOOP PLANT ZEROS

One output	Two output	Three output
0.185		
$-0.598\times10^{-1}\pm j0.411\times10^{-1}$		
-0.519×10^{-1}	-0.619×10^{-1}	-0.619×10^{-1}
-0.154	-0.154	-0.154
-0.185		
-0.214	-0.214	
-0.300		

ultimate value. The initial drop in frequency predicted by the reduced model seems counter-intuitive, but its meaning must be carefully interpreted. Recall, that the reduction process effects an approximation by making the relatively fast dynamics infinitely fast. In this framework the fast dynamics attains a 'tentative' equilibrium before the slow dynamics even begins to respond. This tentative equilibrium produces the drop in frequency which appears to occur instantaneously in the reduced model.

It should also be noted that the approximation becomes precise only as the relative magnitude of the fast eigenvalues becomes large compared to the slow eigenvalues. In this case the boundary is at -0.25 which lies between eigenvalues of -0.224 and -0.274. Thus, it is to be expected that there will be some distortion of the dynamics associated with the fastest of the modes retained in the reduced model. The dynamics of interest for control of course are considerably slower than these modes.

5. GOVERNOR DESIGN

Speed control of steam turbines was classically achieved by the implementation of the fly-ball governor. This was a strictly mechanical apparatus which positioned a steam valve to modulate steam flow, thus effecting turbine speed.

As larger turbine units were designed, the valve and actuating devices changed accordingly (Dinely and Power, 1964) and the mechanical linkage between the governor and the valves was replaced by a hydraulic one. Versions of the mechanical-hydraulic governor system have been used almost exclusively into the early 1960's (Osborne, 1975).

In the mid-1960's, the mechanical actuating mechanism was replaced by analog electronic circuitry. This was necessitated by the increased need for fast closure. The use of mini- and micro-computers led to the development of the Digital Electro-Hydraulic (DEH) governor in the late 1960's. Here, the control logic is handled by digital computer software. DEH was first put into use in the early 1970's (Podolsky, Osborne and Heiser, 1971).

In typical present day installations load control can be carried out by any one of several alternative feedback arrangements at the election of the plant operator. The most elementary structure involves feedforward of the load reference signal to provide a nominal governor valve position which is modified by a term proportional to speed error to provide load participation. An additive manual adjustment is generally available to the operator with this arrangement.

At his discretion the operator may replace the manual adjustment with a proportional plus integral regulator for unit control error. Finally, the operator may elect to employ impulse chamber pressure feedback in which case the previously generated governor valve position is rescaled to represent an impulse chamber pressure set point. A proportional plus integral chamber pressure is used in order to position the governor valves. The use of this last procedure acts to reduce the difficulties which arise because of the highly nonlinear valve characteristics.

The specific controller gains used in the scheme will vary from one application to the next. Evaluation of such gains can be carried out as in (Hope, Malik and Farag, 1976 and Malik, Hope and Farag, 1976).

In this paper, the software flexibility of DEH will be exploited as a vehicle for implementation of a dynamic compensator discussed below. This compensator and the associated design methodology provides a consistent basis for formulating explicit performance objectives and for the evaluation of the effectiveness of using alternative measurement sets for the purpose of meeting a common performance objective.

Performance objectives

In modern, large power generating plants the turbine-generator governing system usually serves two distinct functions. Prior to synchronization the governor is used as a speed control device, and after synchronization it assumes the role of a load control device. The latter is of concern herein. This distinction is not at all subtle. It relates to a change in controller operating mode which is necessitated by the fact that operating considerations are quite different after synchronization than before. First, so long as the machine remains connected with the system its frequency will equilibrate at the average system frequency and deviations from that frequency will tend to be extremely small. Thus, the principal role of the governing system becomes that of matching power output from the generator to a reference load signal.

Speed control after synchronization cannot be ignored, however. If all generators provide precisely their required power output and the outputs of all generator reference load signals sum to exactly the system load at steady state, then the system equilibrium will be achieved at standard frequency. On the other hand, if some units on the system cannot meet their generation requirement or if the load reference signals do not sum precisely to system load then the equilibrium frequency will deviate from standard frequency.

In order to retain regulation of system frequency under such conditions it has become general practice to use a procedure referred to as load participation or frequency bias. This procedure requires modification of the load reference signal by adding to generation error a term proportional to local frequency error. This quantity will be referred to as the 'unit control error' and is denoted by

$$y_1 = \text{unit control error}$$
$$= \Delta\text{PGNEPA} + B \cdot \text{NGNEA}$$

where ΔPGNEPA is the generation error and the constant B is often referred to as the local frequency bias parameter.

The performance specifications for the regulation of unit control error would generally involve both ultimate state and transient criteria. For the purposes of the following analysis the control objectives are:

> The control system is to regulate unit control error to zero under constant disturbances while maintaining closed-loop time constants observable in unit control error to be 20 s or less.

Note that under this specification the open loop modes numbered 1, 2 and 3, although not meeting the speed requirement (since $Re(\lambda) > -0.05$), are of no concern as they are not observable in the unit control error. Modes 4 and 5, however, will have to be controlled to meet the speed requirement.

There are considerations, other than observability in unit control error which might lead to a need to regulate the speed of response of particular dynamic modes. These could relate to the desire to avoid interaction with other control loops or to regulate internal process variables reflecting modes which can be excited by specific system upsets such as a turbine or circulator loop trip. Although such circumstances are excluded from explicit consideration in the performance objective stated above, the analysis which follows could readily be expanded to deal with them.

The design process

The general approach to compensator design taken herein is to make use of the concepts of state variable feedback and dynamic observers. A number of specific techniques for doing so have been proposed. The method used here has been described in Kwatny (1972) and Kwatny, McDonald, and Kalnitsky (1974), and will only be briefly summarized below. Reset action and higher order control modes are included in this method via the artifice of augmenting the system with random bias variables.

The controller design is based on the model:

$$\begin{aligned} \dot{x} &= Ax + Ew + Bu \\ \dot{w} &= Zw + v \\ y &= Cx + Fw + Du \end{aligned} \tag{6}$$

where $\mathbf{x}$ is an n-dimensional state vector, $\mathbf{y}$ is a p-dimensional output vector, $\mathbf{u}$ is an m-dimensional input vector and $\mathbf{w}$ is a q-dimensional random bias vector specifically introduced to characterize external disturbances or model inaccuracies. The bias noise v is a white noise process having zero mean and covariance $V_v\delta(t)$. The limiting case as V_v vanishes is of particular interest.

The design proceeds in three distinct steps: 1) determination of the nominal (or ultimate state) trajectory, 2) design of the state variable feedback controller, and 3) design of the state and bias variable observer. Each step employs a subset of the output equations as follows. The first step employs the output equations

$$y_1 = C_1 x + F_1 w + D_1 u \tag{7}$$

with $r = \dim(y_1) < \dim(u)$. Under appropriate conditions the outputs y_1 will be driven to desired values $\bar{y}_1$ in ultimate state. In the second step, a (possibly) different set of outputs is employed

$$y_2 = C_2 x + F_2 w + D_2 u. \tag{8}$$

The elements of y_2 represent all of those variables which are of concern during the transient. The outputs selected as elements of y_2 and the weights given them in the cost functional (if used) shape the character of the transient behavior of the system. The third step, design of the observer–estimator, utilizes a third subset of the output equations

$$y_3 = C_3 x + F_3 w + D_3 u. \tag{9}$$

The outputs included as elements of the s-dimensional vector $\mathbf{y}_3$ are, of course, only the outputs to be measured. It will be assumed that C_3 and F_3 are of full rank.

The objective is to steer the system so that y_1 tracks the desired value $\bar{y}_1$ while u varies moderately about the nominal value $\bar{u}$. With $\bar{y}_1$ specified, the appropriate values of $\bar{x}$, $\bar{u}$ are obtained by setting $v \equiv 0$ in (6) to obtain the ultimate state equations:

$$\begin{aligned}\dot{\bar{x}} &= A\bar{x}+Ew+B\bar{u}\\ \dot{w} &= Zw\\ \bar{y}_1 &= C\bar{x}+Fw+D\bar{u}.\end{aligned}\tag{10}$$

Solutions of the form

$$\begin{aligned}\bar{x}&=X_1\bar{y}_1+X_2w\\ \bar{u}&=U_1\bar{y}_1+U_2w\end{aligned}\tag{11}$$

are frequenctly obtainable and are sought by direct substitution in (10).

The state variable feedback gain matrix K can be selected by any of several procedures such as minimization of a quadratic performance index or by pole shifting. The control is then given by

$$u_1=-M\hat{x}_1, M=[K_{\prime}'-KX_2-U_2]$$

where

$$u_1\triangleq u-U_1\bar{y}_1, x_1\triangleq\begin{pmatrix}x-X_1y_1\\ w\end{pmatrix}\tag{12}$$

and $\hat{x}_1$ is an estimate of x_1 as defined below.

The estimate $\hat{x}_1$ is given by

$$\hat{x}_1=H^*(y_3-D_3u_1)+\theta_2\xi\tag{13a}$$

$$\dot{\xi}=(\Lambda A_1\theta_2)\xi+(\Lambda B_1)u_1+(\Lambda A_1H^*)(y_3-D_3u_1)\tag{13b}$$

$$\bar{y}_3=[C_3X_1+D_3U_1]\bar{y}_1$$

where the parameters are defined below. The following matrices are introduced.

$$A_1=\begin{pmatrix}A & E\\ 0 & Z\end{pmatrix}, H=\Bigg[\underbrace{H_1}_{n+q-s}{}' \underbrace{H_2}_{s}\Bigg]=C_3{}'F_3\Bigg],$$

$$G=\begin{pmatrix}0_{nxq}I_q\end{pmatrix}, B_1=\begin{pmatrix}B\\ \hline 0_{qxm}\end{pmatrix}$$

$$\theta_2=\begin{pmatrix}I_{n+q-s}-H_{01}^*H_1\\ \hline -H_{02}^*H_1\end{pmatrix}\tag{14}$$

$$H_0^*=\begin{pmatrix}H_{01}^*\\ \hline H_{02}^*\end{pmatrix}\begin{matrix}\updownarrow n+q-s\\ \updownarrow s\end{matrix}=(CV_vG')H'(F_3V_2F_3')^{-1}$$

$$\Lambda_0=(I_{n+q-s}-H_{01}^*\,|\,-H_{01}^*H_2),$$

H^* can be defined as follows. Select a state feedback gain matrix S, by any means, which stabilizes the system

$$\dot{z}=(\Lambda_0A_1\theta_2)'z+(HA_1\theta_2)'v,\quad v=-Sz.$$

Then,

$$H^*=H_0^*+\theta_2S'.\tag{15}$$

and

$$A=(I_{n+q-s}-H_1^*H_1\,|\,-H_2^*H_2).\tag{16}$$

The $2n+q$ eigenvalues of the closed-loop system include the q eigenvalues of Z associated with the bias variables w, the n stable eigenvalues corresponding to the closed-loop system matrix $A=A-BK$, and the $n+q-s$ eigenvalues of the observer matrix $\Lambda A_1\theta_2$.

When (12) and (13) are resolved the equivalent compensator transfer relation is found to be

$$\begin{aligned}U_1(s)=&-\{I|sI-\Delta|+[I-MH^*D_3]^{-1}\\ &\cdot M\theta_2\mathrm{Adj}\,(sI-\Delta)\Omega\}^{-1}[I-MH^*D_3]^{-1}\\ &\cdot M\{I|sI-\Delta|+\theta_2\mathrm{Adj}\,(sI-\Delta)\Lambda A_1\}H^*\bar{Y}_3(s)\end{aligned}\tag{17}$$

where

$$\Delta=\Lambda A_1\theta_2, \Omega=\Lambda B_1-\Lambda A_1H^*D_3.$$

Comparison of alternative designs

The procedure described above has been used to design three governors for the turbine-generator system, having access to, respectively, one, two and three plant measurements as follows:

Single input–single output (SISO): the only measurement is unit control error which is the minimum requirement if the ultimate state objective is to be met.

Single input–dual output (SIDO): the measurements are frequency and generation.

Single input–triple output (SITO): the measurements are frequency, generation and impulse chamber pressure.

In each case the regulator is designed to meet the common performance objective and is based on the reduced linear model (8th order) defined above. Thus, in each case the compensator itself is 8th order. Consequently, in each case there are 16 closed loop eigenvalues. Moreover, these are the same for each design and they are listed in Table 6. Note that eight are associated with the

TABLE 6. CLOSED LOOP EIGENVALUES

State variable feedback			
-0.619×10^{-1}		-0.619×10^{-1}	
-0.621×10^{-1}	$\pm j0.119\times10^{-1}$	-0.621×10^{-1}	$\pm j0.119\times10^{-1}$
-0.132	$\pm j0.804\times10^{-2}$	-0.132	$\pm j0.804\times10^{-2}$
-0.212		-0.212	
-0.224		-0.224	

state-variable feedback system and eight with the observer. Also, because of the design strategy these assume the same values. Observe that only the eigenvalue corresponding to the mode which violates the performance objective has been moved.

Useful insight can be obtained by examining the compensator form of the controller. The Tables 7 and 8 contain the compensator poles and zeros, respectively, for each of the three designs.

considered here the specifications are such that it is possible to meet them with one, two or three measurements. If the dynamic specifications were such that it was necessary to meet a 15 s requirement (rather than 20 s) it would have been impossible to do so with any of the alternative measurement sets because of controllability problems with mode 6. In fact, the only observability problem which obtains relief by addition of the third measurement is mode 10. Thus, it would seem that there is little advantage to be gained by the measurement of impulse chamber pressure.

TABLE 7. COMPENSATOR POLES

SISO	SIDO	SITO
0.0	0.0	0.0
−0.0619	−0.0619	−0.0619
−0.0630±j0.0463	−0.0610±j0.0355	−0.0602±j0.319
−0.154	−0.154	−0.154
−0.182	−0.182	−0.194
−0.214	−0.214	−0.220±j0.0181
−0.294	−0.294	

TABLE 8. COMPENSATOR ZEROS

SISO	SIDO	SITO
−0.0489±j0.6111		
−0.0619	−0.0619	−0.0619
−0.132±j0.00804	−0.132±j0.00804	−0.132±j0.00804
−0.154	−0.154	−0.154
−0.212	−0.212	−0.212
−0.224	−0.224	−0.224

There are several observations worth noting. First observe that in all cases the compensator has a single pole at the origin which provides for the requirement of zero unit control error in steady state. Second, a form of pole-zero cancellation obtains and in each case the compensator contains zeros which cancel those plant poles which satisfy the dynamic performance requirements. Third, recall that modes 6, 9 and 10 are either essentially uncontrollable or unobservable or both in the one and two output cases and consequently pole zero cancellation occurs in the plant transfer matrix. It is seen from Tables 7 and 8 that cancellation also occurs in the compensator, illustrating that these modes could have been removed from the model for the purposes of regulator design resulting in a 5th order compensator. Note that in the case of three output measurements this statement is true for modes 6 and 9 only.

One important question is whether there is any advantage to the multiple measurement compensators and if so, what are they? In the design

This erroneous view is further supported by the following considerations. Consider the *closed loop* response of unit control error to a change in generation demand. This closed loop system can be viewed as a single input single output system regardless of which of the three compensators are used. In each case, the closed loop eigenvalues are the same and are listed in Table 6. The closed loop zeros can be calculated using each compensator and again they turn out to be the same. They are, in fact, those values listed in the first column of Table 5 and the first column of Table 8. Consequently, regardless of which compensator is used the transfer function between generation demand and unit control error is the same.

These results are based on the use of the reduced linear model. In order to further explore their validity each compensator was simulated with the full state nonlinear plant model. Figure 2 illustrates generation response for a 5% step increase in generation demand, for the one and

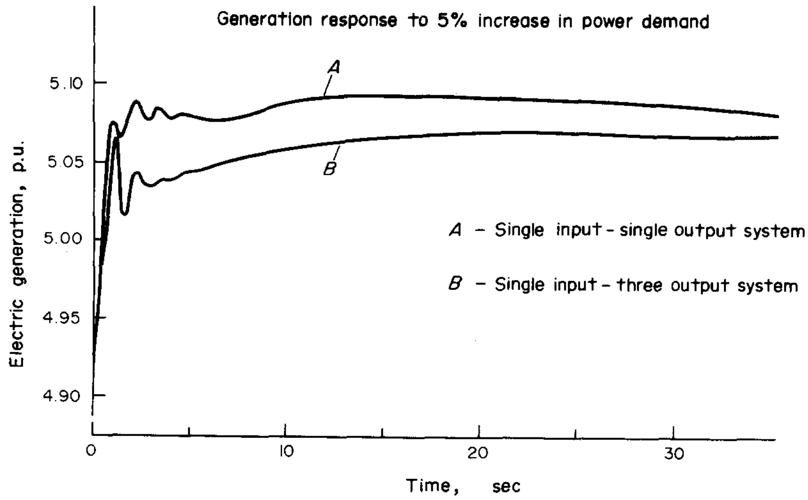

FIG. 2. Generation response to 5% increase in power demand.

three measurement compensators. Although they are similar, they are certainly not identical. These differences must be attributable to the fact that the considerable nonlinearities in the simulated model are exerting their influence.

Since the differences between the full state nonlinear simulation and the reduced linear model used for design can be viewed as disturbances acting on the approximate model it is interesting to examine the poles and zeros of the closed loop transfer matrix between the disturbance inputs and the unit control error for each of the compensators. Again, the poles remain unchanged regardless of the compensator employed. The zeros are different in each case, however. Thus, it is expected that the system will respond differently to disturbances (or model inaccuracies) depending upon which of the compensators is employed.

An interesting operating procedure which illustrates this behavior is a change in valve configuration. With modern electro-hydraulic governing systems the plant operator is frequently provided with the ability to change the sequence in which the multiple turbine governing values are opened. Of particular interest is the ability to change from partial arc admission (sequential valve operation) to full arc admission (all valves are opened in parallel). The ability to perform such maneuvers considerably increases plant operating flexibility. The switch from partial to full arc admission represents a very dramatic change in the valve characteristic. In general, the valve stroke required to yield the same throttle flow will be quite different even though steam conditions at the throttle are the same. In practice, the computer control system would have sufficient information about the valve characteristic in either mode of operation so that the feedforward signal for valve stroke would be modified in order to minimize the disturbance.

Figure 3 illustrates generation response to a valve configuration change from partial to full arc operation. The feedforward signal is not modified so that the system disturbance is maximized for illustrative purposes. It is necessary for the feedback system to correct for any model inaccuracies. Clearly this illustrates the advantage of impulse chamber pressure measurement from the viewpoint of disturbance accommodation.

6. CONCLUSIONS

This paper represents an early step in the analysis of dual turbine-generator installations. Heretofore, most analyses of such systems have been confined to simulation studies. The work reported herein is intended to supplement such studies and brings to bear well established techniques of linearization and multivariable linear systems analysis. Of the several alternative operating alignments of interest, the numerical details of only one are presented in this paper. Others are under study at this time.

A modal analysis shows that there are several modes involving thermodynamic variables which are sufficiently slow to be candidates for regulation by the digital governor. Some of these are not observable in unit control error and hence are of no interest in terms of existing criteria for governor design.

The calculation of system zeros proved to be a valuable adjunct to controllability–observability analysis. The success of the procedure of diagonalizing the A matrix and examining the transformed B and C matrix for zero rows and columns

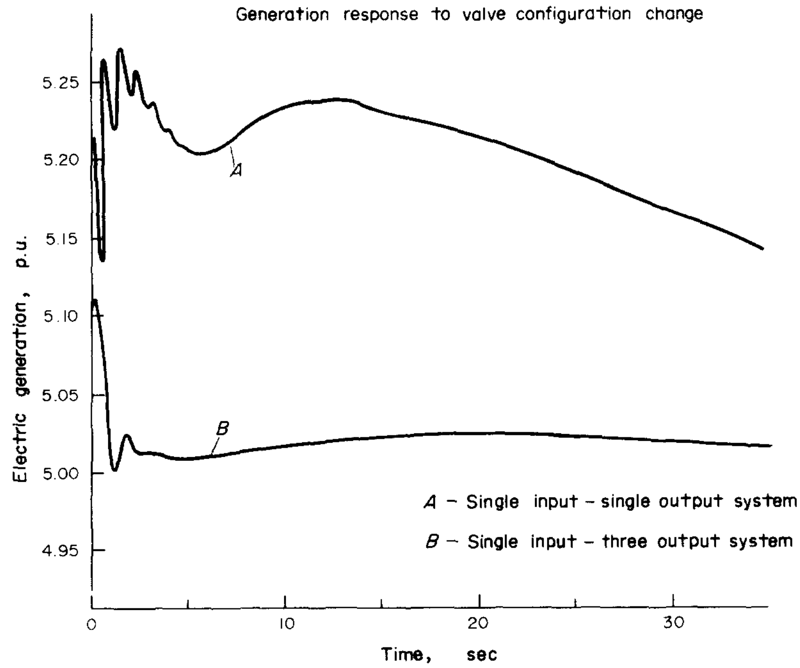

FIG. 3. Generation response to valve configuration change.

respectively is very highly dependent upon scaling. In dealing with large scale physical processes determining ideal scaling for system variables is a significant problem. On the other hand, pole-zero cancellation (or approximate cancellation) which occurs when there is a controllability and/or observability problem appears to be less sensitive to scaling and can be used to confirm or support such conclusions.

The importance of the measurement of impulse chamber pressure in terms of disturbance accommodation has been demonstrated. This confirms experience. Attempts in the field to affect an automatic transfer from partial arc to full arc admission or vice-versa without incorporating this measurement have proved to be unsatisfactory.

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APPENDIX A: PARAMETER VALUES FOR LINEAR STATE MODEL (NONZERO ELEMENTS ONLY)

A Matrix

A(1,1) = −0.2674 E02	A(1,2) = −0.1199 E00	A(1,3) = 0.8662 E01
A(2,1) = −0.1526 E04	A(2,2) = −0.2231 E02	A(1,3) = 0.5708 E03
A(3,1) = 0.1208 E01	A(3,2) = 0.5417 E−02	A(3,3) = −0.1847 E03
A(4,1) = −0.1359 E03	A(4,2) = −0.2475 E01	A(4,3) = 0.5544 E02
A(5,1) = −0.4577 E02	A(5,2) = −0.8287 E00	A(5,3) = 0.3470 E02
A(6,1) = 0.4192 E00	A(6,2) = 0.1882 E−02	A(6,3) = −0.7067 E00
A(15,1) = 0.3159 E−01	A(15,2) = 0.6849 E−04	A(7,3) = 0.8267 E01
A(16,1) = 0.4384 E−01	A(16,2) = 0.2681 E−04	A(8,3) = −0.1851 E01
A(21,1) = 0.1824 E−01	A(21,2) = 0.3954 E−04	A(9,3) = 0.8263 E01
A(25,1) = 0.1020 E−03	A(25,2) = 0.4571 E−06	A(10,3) = −0.4396 E01
A(26,1) = −0.1343 E−01	A(26,2) = 0.7075 E−04	A(15,3) = 0.2860 E−01
A(27,1) = −0.4251 E−01	A(27,2) = −0.1905 E−02	A(16,3) = 0.2540 E−04
A(34,1) = 0.1022 E02	A(34,2) = 0.2739 E00	A(34,3) = −0.8326 E01

A(1,4) = 0.1995 E−01	A(3,5) = 0.8688 E−03	A(3,6) = 0.8694 E−03
A(2,4) = 0.1319 E01	A(4,5) = −0.6844 E00	A(4,6) = −0.6943 E00
A(3,4) = −0.4256 E00	A(5,5) = −0.4598 E00	A(6,6) = −0.1384 E01
A(4,4) = 0.1313 E00	A(6,5) = 0.1386 E01	A(7,6) = −0.7805 E−04
A(5,4) = 0.7940 E−01	A(7,5) = −0.7800 E−04	A(10,6) = 0.6188 E−01
A(6,4) = −0.1627 E−02	A(8,5) = 0.6183 E−01	A(15,6) = 0.2540 E−04
A(7,4) = 0.1905 E−01	A(15,5) = 0.6489 E−04	A(16,6) = 0.4677 E−04
A(8,4) = −0.4265 E−02	A(16,5) = 0.2540 E−04	A(21,6) = 0.1467 E−04
A(9,4) = 0.1904 E−01	A(21,5) = 0.3746 E−04	
A(10,4) = −0.1013 E−01		
A(15,4) = 0.4677 E−04		
A(16,4) = 0.6491 E−04		
A(21,4) = 0.2700 E−04		
A(34,4) = −0.1919 E−01		

A(3,7) = 0.9207 E02	A(3,8) = 0.2105 E00	A(3,9) = 0.9202 E02
A(4,7) = 0.3953 E02	A(4,8) = 0.8981 E−01	A(4,9) = 0.7230 E01
A(6,7) = 0.3199 E02	A(6,8) = 0.7312 E−01	A(6,9) = −0.3197 E02
A(7,7) = −0.8404 E01	A(7,8) = −0.1914 E−01	A(9,9) = −0.8399 E01
A(8,7) = 0.1865 E01	A(8,8) = −0.5757 E−01	A(10,9) = 0.4428 E01
A(11,7) = 0.2145 E00	A(11,8) = 0.1546 E00	A(13,9) = 0.2000 E00
A(12,7) = 0.2559 E00	A(12,8) = 0.4572 E−03	A(14,9) = 0.2560 E00
A(15,7) = 0.5292 E−01	A(15,8) = 0.6487 E−04	A(15,9) = 0.2874 E−01
A(16,7) = 0.7344 E−01	A(16,8) = 0.2539 E−04	A(16,9) = 0.5292 E−01
A(17,7) = 0.7031 E00	A(17,8) = 0.1179 E00	A(18,9) = 0.6922 E00
A(19,7) = −0.6243 E02	A(19,8) = −0.1097 E00	A(21,9) = 0.4659 E−02
A(20,7) = 0.1198 E01	A(20,8) = 0.2088 E−02	A(22,9) = −0.6344 E02
A(21,7) = 0.3055 E−01	A(21,8) = 0.3745 E−04	A(23,9) = 0.1201 E01

A(3,10) = 0.2104 E00	A(7,11) = 0.1141 E−04	A(7,12) = 0.6877 E−01
A(4,10) = 0.1673 E−01	A(11,11) = −0.1544 E00	A(11,12) = −0.2972 E00
A(6,10) = −0.7308 E−01	A(12,11) = −0.2911 E−02	A(12,12) = −0.1821 E01
A(9,10) = −0.1913 E−01	A(15,11) = 0.6769 E01	A(15,12) = 0.4113 E04
A(10,10) = −0.5184 E−01	A(16,11) = 0.2541 E−04	A(16,12) = 0.7335 E−01
A(13,10) = 0.1546 E00	A(17,11) = 0.1564 E00	A(17,12) = −0.1909 E02
A(14,10) = 0.4572 E−03	A(19,11) = −0.1120 E01	A(19,12) = −0.7080 E03
A(15,10) = 0.4675 E−04	A(20,11) = 0.2127 E−01	A(20,12) = 0.1345 E02
A(16,10) = 0.6488 E−04	A(21,11) = 0.3747 E−04	A(21,12) = 0.2300 E−01
A(18,10) = 0.1180 E00		
A(21,10) = 0.2700 E−04		
A(22,10) = −0.1098 E00		
A(23,10) = 0.2088 E−02		

A(9,13) = 0.1141 E−03	A(9,14) = 0.6866 E−01	A(15,15) = −0.1459 E01
A(13,13) = −0.1544 E00	A(13,14) = −0.2777 E00	A(16,15) = −0.2097 E−01
A(14,13) = −0.2909 E−02	A(14,14) = −0.1831 E01	A(17,15) = −0.1012 E−02
A(15,13) = 0.4677 E−04	A(15,14) = 0.1018 E00	A(21,15) = 0.4088 E−01
A(16,13) = 0.6770 E01	A(16,14) = 0.4087 E04	A(24,15) = −0.8458 E−02
A(18,13) = 0.1563 E00	A(18,14) = −0.1897 E02	
A(21,13) = 0.2700 E−04	A(21,14) = 0.5877 E−01	
A(22,13) = −0.1120 E01	A(22,14) = −0.7080 E03	
A(23,13) = 0.2125 E−01	A(23,14) = 0.1345 E02	

A(15,16) = -0.2112 E-01
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A(12,17) = -0.1989 E-03
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A(15,21) = 0.6424 E-05
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A(21,26) = 0.6659 E00
A(22,26) = -0.2267 E01
A(23,26) = 0.7184 E-01
A(24,26) = 0.6658 E00
A(25,26) = -0.3639 E-02
A(26,26) = -0.1060 E-02
A(27,26) = -0.8821 E-03
A(28,26) = -0.8744 E-03
A(30,26) = 0.4386 E-01
A(31,26) = 0.1359 E02

A(15,27) = 0.2232 E-04
A(16,27) = 0.4109 E-04
A(21,27) = 0.3618 E-06
A(25,27) = 0.3812 E-04
A(26,27) = -0.1028 E00
A(27,27) = -0.2065 E00
A(30,27) = -0.1174 E-02
A(31,27) = 0.2880 E01

A(15,28) = 0.4109 E-04
A(16,28) = 0.5702 E-04
A(21,28) = 0.2372 E-04
A(27,28) = 0.1027 E00
A(28,28) = -0.1026 E00

A(15,29) = 0.8769 E-01
A(16,29) = 0.9636 E-02
A(21,29) = 0.5062 E-01
A(26,29) = -0.2779 E-01
A(26,29) = -0.1048 E00
A(27,29) = 0.7367 E-03
A(30,29) = 0.8543 E00
A(31,29) = 0.2649 E-03

A(15,30) = 0.6604 E-01
A(16,30) = 0.1216 E00
A(21,30) = 0.1107 E-01
A(25,30) = 0.1907 E01
A(26,30) = 0.8650 E-01
A(27,30) = -0.5059 E-01
A(30,30) = -0.6225 E02
A(31,30) = -0.1928 E05
A(32,30) = 0.1407 E01
A(33,30) = 0.6038 E03
A(34,30) = 0.4530 E02

A(15,31) = 0.4590 E-04
A(16,31) = 0.6369 E-04
A(21,31) = 0.2650 E-04
A(25,31) = 0.1705 E-02
A(26,31) = 0.4244 E-04
A(27,31) = -0.4523 E-04
A(30,31) = -0.5565 E-01
A(31,31) = -0.2139 E02
A(32,31) = 0.1257 E-02
A(33,31) = 0.3734 E01
A(34,31) = 0.7497 E-01

A(15,32) = 0.4747 E00
A(16,32) = 0.1858 E00
A(21,32) = 0.2740 E00
A(30,32) = 0.1080 E01
A(31,32) = 0.3315 E03
A(32,32) = -0.3578 E01
A(33,32) = -0.1135 E04
A(34,32) = 0.4800 E02

A(15,33) = 0.2653 E-04
A(16,33) = 0.4884 E-04
A(21,33) = 0.4300 E-05
A(30,33) = 0.4239 E-03
A(31,33) = 0.1314 E00
A(32,33) = -0.1407 E-02
A(33,33) = -0.4591 E01
A(34,33) = 0.1614 E00

A(15, 34) = 0.3786 E01
A(16, 34) = 0.5255 E01
A(21, 34) = 0.2186 E01
A(34, 34) = −0.1049 E01
A(35, 34) = 0.9993 E00

A(15, 35) = 0.7929 E−01
A(16, 35) = 0.8714 E−02
A(21, 35) = 0.4578 E−01
A(34, 35) = −0.2165 E02
A(36, 35) = 0.5218 E02

A(15, 36) = 0.5331 E−02
A(16, 36) = 0.3495 E−01
A(21, 36) = 0.1096 E−01
A(36, 36) = −0.2981 E02
A(37, 36) = 0.8004 E−01
A(38, 36) = 0.1768 E00

A(15, 37) = 0.4366 E00
A(16, 37) = 0.6060 E00
A(21, 37) = 0.2521 E00
A(36, 37) = 0.3538 E03
A(37, 37) = −0.9995 E00

A(15, 38) = 0.4843 E−01
A(16, 38) = 0.1896 E−01
A(21, 38) = 0.2796 E−01
A(35, 38) = −0.2384 E02
A(36, 38) = −0.2098 E02
A(38, 38) = −0.1764 E00

B Matrix

B(2, 1) = 0.1974 E04
B(3, 1) = −0.3068 E−02
B(4, 1) = 0.6422 E00
B(5, 1) = 0.5242 E−04
B(6, 1) = −0.1043 E−02
B(15, 1) = 0.3432 E−02
B(16, 1) = 0.6318 E−02
B(21, 1) = 0.5563 E−03
B(25, 1) = −0.3411 E−04
B(26, 1) = 0.1366 E−02
B(27, 1) = 0.1421 E−01
B(34, 1) = 0.2141 E00

C Matrix

C(3, 1) = 0.7085 E03
C(3, 2) = 0.3173 E01
C(1, 34) = 0.9994 E00
C(4, 34) = 0.9994 E00
C(2, 35) = 0.2474 E01
C(2, 38) = 0.2725 E01

D Matrix

0